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Three-dimensional instability in compressible reacting mixing layers with heat release †

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Abstract

This paper investigates the linear stability of compressible reacting mixing layers with special emphasis on three-dimensionality. The governing equations for laminar flows are from two-dimensional compressible boundary-layer equations. The chemistry is a finite rate single step irreversible reaction with Arrhenius kinetics. For incompressible reacting mixing layers, two-dimensional outer modes are more amplified than three-dimensional ones. For compressible non-reacting mixing layers at $M_c > 0.6$, the most unstable modes are oblique center modes that are subsonic relative to both free streams. For compressible reacting mixing layers with $T_{ad} > 3$, the most unstable modes are two-dimensional outer modes even at high Mach numbers. Three-dimensional modes agree well with experimental data compared to two-dimensional modes.

Keywords**:** At least four keywords; Compressible mixing layer; Instability; Reacting shear flow; Three-dimensionality

1. Introduction

Reacting free shear layers occur in many systems, including gas turbine combustors and rockets. The large temperature change caused by the combustion alters the thermodynamic properties of the flow considerably and produces dilatation; both of these effects cause a strong interaction between the hydrodynamics and the chemistry, making the study of reacting flows difficult.

For incompressible parallel inviscid flow, Rayleigh [1] showed that if the velocity profile has an inflection point the flow is unstable. Due to the increasing importance of high-speed flows, Lees and Lin [2] investigated the stability of compressible mixing layers to infinitesimal disturbances and classified the disturbances as subsonic, sonic and supersonic. Lessen et al. [3] suggested that the three-dimensional waves are more unstable than two-dimensional ones at high Mach numbers. Gropengiesser [4] carried out inviscid spatial stability calculations and found a second mode of instability to three-dimensional disturbances at high Mach numbers and that the most unstable modes are three-dimensional at high Mach numbers. Sandham and Reynolds [5] solved the linearized inviscid compressible stability equation and found that three-dimensional effects are important at high Mach numbers.

Previous studies showed that the three-dimensional waves are more unstable than the two-dimensional waves at high numbers in non-reacting compressible mixing layers, but the effect of compressibility in reacting mixing layers still raises many questions. Therefore, this paper studies the three-dimensional linear instability characteristics in the compressible reacting plane shear layer in which fuel and oxidizer are initially unmixed. The simplest model of a reacting mixing layer involving two uniform parallel

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streams containing the fuel and oxidizer is used as a laminar flow. The chemistry is a finite rate single step irreversible reaction with Arrhenius kinetics. The governing equations for laminar flows are the boundary-layer equations for the two-dimensional compressible flow.

2. Laminar flow profiles and disturbance equations

Laminar flow profiles and disturbance equations are almost the same as in our previous paper [6], and only the differences from the compressible characteristics will be discussed. We assume that the fast stream contains the fuel and the slow stream, the oxidizer; U_1 and U_2 represent the free-stream velocities.

The governing equations for stability analysis are Euler equations for the compressible ideal gas including chemical reactions. To simplify the stability analysis, we assume that the laminar flow is parallel, i.e., that its variation is entirely in the direction normal to the flow. All flow variables can be considered as the sum of the mean flows and the wave-like disturbances.

$$f(x,y,z,t) = \bar{f}(y) + f'(x,y,z,t) \tag{1}$$

$$f'(x,y,z,t) = \hat{f}(y)\exp\left[i(\alpha x + \beta z - \omega t)\right] \tag{2}$$

$\bar{f}(y)$ is the laminar profile of a quantity and $\hat{f}(y)$ is an eigenfunction assumed to be a function of y coordinate only, and α and β are wave numbers in the streamwise and spanwise directions, respectively, and ω is the frequency. The relation between the wave numbers and the angle of disturbance is

$$\tan\theta = \beta/\alpha \tag{3}$$

For the temporal stability analysis, α is real and ω is complex, whereas for the spatial analysis, ω is real and α is complex. The amplification rates for the two cases are ω_i and $-\alpha_i$, respectively. The complex wave velocity c is ω/α.

Substituting Eq. (2) into Euler equations and neglecting products of perturbations, we obtain the following linearized equations for the pressure:

$$\begin{aligned}&\hat{p}'' - \left\{\frac{2\alpha\bar{u}'}{(\alpha\bar{u}-\omega)} + \frac{\bar{\rho}}{\bar{T}}(\alpha\bar{u}-\omega)^2[RXN1]\right\}\hat{p}' - \\ &\left[\left(\alpha^2+\beta^2\right) - \gamma M_1^2(\alpha\bar{u}-\omega)^2 \times \left\{\frac{1}{\bar{T}} + \frac{\bar{\rho}}{\bar{T}}[RXN2]\right\}\right]\hat{p} = 0\end{aligned} \tag{4}$$

where prime indicates differentiation with respect to y. The terms $[RXN1]$ and $[RXN2]$ are the terms that represent the effect of density variation due to chemical reaction and compressibility on the instability.

The boundary conditions are obtained by considering the asymptotic form of the solutions of Eq. (4). As $y \to \pm\infty$, $\bar{u}'$ and $[RXN1]$ become negligible and $[RXN2]$ becomes $-\bar{T}(\gamma-1)/\gamma$. Eq. (4) then reduces to

$$\hat{p}'' - q^2\hat{p} = 0 \tag{5}$$

$$q^2 = \left(\alpha^2+\beta^2\right) - \frac{M_1^2(\alpha\bar{u}-\omega)^2}{\bar{T}} \tag{6}$$

As $y \to \pm\infty$, the pressure must behave like

$$\hat{p} \to \exp(\pm qy) \quad \text{as} \quad y \to \mp\infty \tag{7}$$

The nature of the disturbances can be described in terms of the relative Mach numbers, M_{ri}, which are defined as the Mach numbers of the disturbances in the direction of the wavevector (α,β) relative to the free-streams i.

$$\begin{aligned}M_{r1} &= \frac{\alpha\left(c^* - \bar{u}_1^*\right)}{\left(\alpha^2+\beta^2\right)^{1/2} a_1^*} = \tilde{M}_1(c-1) \\ M_{r2} &= \frac{\alpha\left(c^* - \bar{u}_2^*\right)}{\left(\alpha^2+\beta^2\right)^{1/2} a_2^*} = \frac{\tilde{M}_1(c-\bar{u}_2)}{\bar{T}_2^{1/2}}\end{aligned} \tag{8}$$

The superscript * represents the dimensional quantity, c^* is the phase velocity of the disturbance and a_i^* is the sonic speed in the free-stream i. When the magnitude of a relative Mach number is less than unity, the instability wave is said to be subsonic with respect to that boundary; when it is greater than unity, it is said to be supersonic with respect to that boundary. At the zeros of q^2, the instability waves are sonic ($|M_r| = 1$). We define c_u as the phase speed of a disturbance that is sonic with respect to the upper stream and c_l as the corresponding speed with respect to the lower stream:

$$\begin{aligned}c_l &= \bar{u}_2 + \frac{\left(\alpha^2+\beta^2\right)^{1/2}\bar{T}_2^{1/2}}{\alpha M_1} = \bar{u}_2 + \frac{\bar{T}_2^{1/2}}{M_1} \\ c_u &= 1 - \frac{\left(\alpha^2+\beta^2\right)^{1/2}}{\alpha M_1} = 1 - \frac{1}{\tilde{M}_1}\end{aligned} \tag{9}$$

The relative Mach number of the most unstable mode was called the convective Mach number by Mack [7].

An iterative method based on the shooting and Newton-Raphson method is used to solve Eq. (4).

3. Three-dimensional Instability

Shin et al. [6] showed that a necessary condition for instability of the incompressible reacting mixing layer is that $\left(\overline{\rho u'}\right)'$ changes sign at least once in the flow domain. They showed that the mean profiles for reacting flow change sign three times and should have three independent modes of instability. The first mode is the center mode that arises from the central inflection point; its phase velocity is the mean velocity at the central inflection point. The second represents both of the modes due to the outer inflection points. The phase velocities of the two outer modes are different and they approach the mean velocities at the outer inflection points as the wave number increases. The center mode travels at the same phase velocity as in the cold flow, the average of the two free-stream velocities. One of the outer modes travels at lower speed than the center mode, whereas the outer travels at higher speed. They reported that for the large heat releases typical of combusting flows, the outer mode is more amplified than the center mode.

Squire's theorem [8] states that the lowest Reynolds number for transition occurs when the disturbances are two-dimensional, so two-dimensional modes dominate the viscous instability of incompressible flows. However, the most unstable modes are three-dimensional for a compressible non-reacting mixing layer. We tried to determine whether the most amplified modes are two- or three-dimensional in the incompressible low-speed reacting mixing shear layer. Fig. 1 shows the temporal maximum growth rates of three different oblique angles ($\theta = 0,30,60°$). T_{ad} is non-dimensional flame temperature to express the amount of heat release. The Damköhler number, Da, is the ratio of a characteristic flow time to a characteristic chemical reaction time. The first part of the curve corresponds to the instability of the center mode and the second part to the one of the outer modes. When the heat release is large, the outer modes are more unstable than the center mode. The disturbances become more stable and the corresponding wave numbers smaller as the obliquity increases. Fig. 2 shows the maximum growth rates as a function of oblique angle. The outer modes are more unstable and that

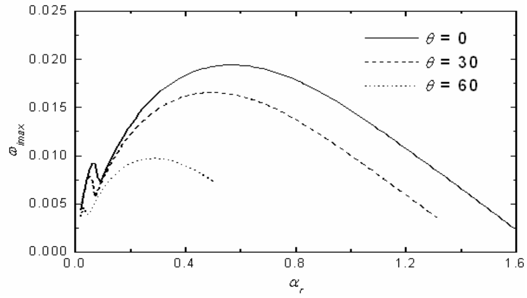

Fig. 1. Growth rates of three different oblique angles ($\theta = 0,30,60°$) in temporal stability. $M_1 = 0, T_{ad} = 8, \overline{T}_2 = 1, Da = 10$.

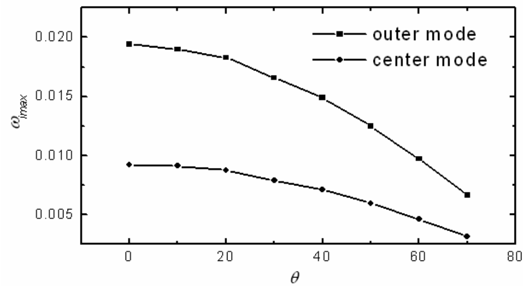

Fig. 2. Growth rates versus oblique angle (temporal stability). $M_1 = 0, T_{ad} = 8, \overline{T}_2 = 1, Da = 10$.

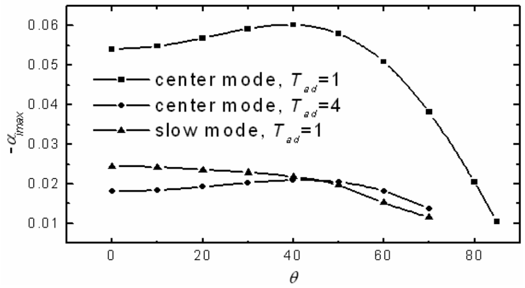

Fig. 3. Maximum growth rates versus oblique angle (spatial stability). $M_1 = 3(M_c = 0.75), \overline{T}_2 = 1, Da = 10$.

they become more stable as the obliquity increases. It suggests that the most unstable modes are two-dimensional in incompressible reacting flows.

Sandham and Reynolds [5] reported that the most unstable mode in compressible flows becomes three-dimensional when the convective Mach number, M_c, is larger than 0.6. We study the effect of heat release ($T_{ad} = 4$) on the obliquity of the most amplified mode in high speed flows ($M_1 = 3.5$). Fig. 3 shows the spatial maximum growth rates as a function of oblique angle at convectively subsonic flows ($M_1 = 3.5, M_c = 0.75$). Only the slow mode growth rates are given since the outer modes have almost the same growth rates. In a non-reacting mixing layer, the most unstable mode is the oblique center mode ($\theta = 37°$) and its growth rate is about 11% higher than the corresponding two-dimensional mode. Sandham and Reynolds [5] found that for the most amplified disturbance;

$$M_c \cos\theta \approx 0.6 \quad (10)$$

This relation predicts the most unstable mode to be at $37°$ for $M_c = 0.75$, which is very close to the current result. At $T_{ad} = 4$, the outer modes are more unstable than the center mode. The outer modes become more stable but the center mode unstable as the obliquity increases. The oblique angle for the most unstable center mode is about $50°$. However, the growth rate of the most unstable oblique center mode is about 20% lower than the two-dimensional outer modes and the most unstable modes are two-dimensional when considerable heat release exists.

Fig. 4 shows the spatial maximum growth rates as a function of oblique angle at convectively supersonic flows ($M_1 = 5, M_c = 1.25$). In non-reacting mixing layer, for angles less than about $37°$ the outer modes are dominant and the maximum growth rate changes little with angle. They are supersonic unstable modes that have no connection with the generalized inflection points. However, when the angle is greater than $37°$, the outer modes disappear and the center mode begins to dominate. The reason for the transition from outer mode dominance to center mode dominance can be understood by examining the relative Mach number, M_r in Eq. (8). For waves propagating at angle θ relative to the x direction, Eqs. (8) and (9) become

$$M_{r1} = M_1(c-1)\cos\theta, \quad M_{r2} = \frac{M_1(c-\bar{u}_2)\cos\theta}{\bar{T}_2^{1/2}} \quad (11)$$

$$c_l = \bar{u}_2 + \frac{\bar{T}_2^{1/2}}{\tilde{M}_1}, \quad c_u = 1 - \frac{\left(\alpha^2+\beta^2\right)^{1/2}}{\alpha M_1} = 1 - \frac{1}{\tilde{M}_1} \quad (12)$$

Fig. 5 shows the $c_{l,u}$ for $M_1 = 5$. Because the relative Mach numbers are functions of the propagation angle, there is a possibility of transition from supersonic (regions II, III, and IV) to subsonic (region I) disturbances as the angle increases. $c_l = c_u$ defines the smallest transition angle and , for $M_1 = 5$, it is about $37°$. The non-reacting center modes in Fig. 5 above $37°$ are subsonic relative to both free streams and have the same characteristics as in incompressible flows. For $M_c = 1.25$ ($M_1 = 5$), Eq. (10) suggests that the angle of the most unstable mode is $62°$, which is very close to the angle of maximum instability of the center mode ($66°$). The relative Mach number for the most unstable center mode is 0.53 and is subsonic. Note that the maximum growth rate of oblique center modes is much greater

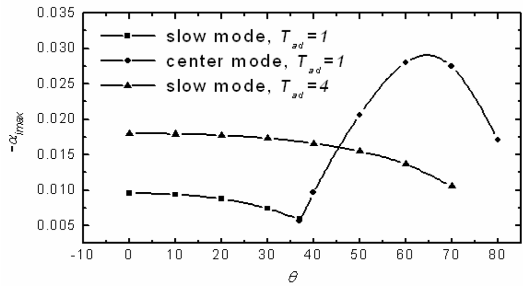

Fig. 4. Maximum growth rates versus oblique angle (spatial stability). $M_1 = 5(M_c = 1.25), \bar{T}_2 = 1, Da = 10$.

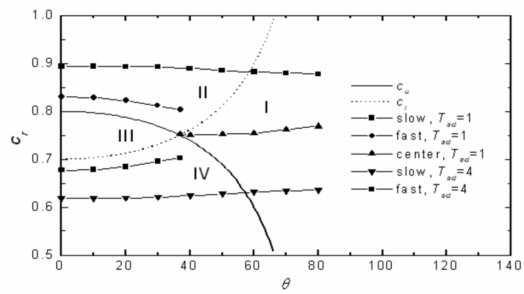

Fig. 5. Phase velocity of maximum growth rates versus oblique angle (spatial stability). $M_1 = 5(M_c = 1.25), \bar{T}_2 = 1, Da = 10$.

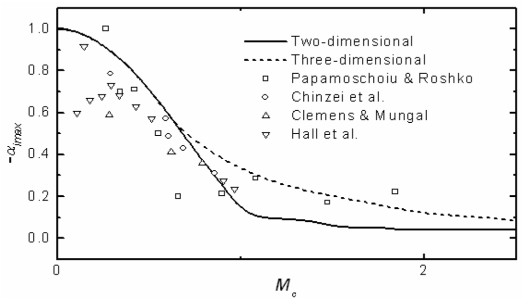

Fig. 6. Comparison of normalized growth rates between theory and experiments.

than the growth rate of two-dimensional outer modes. Therefore, the most unstable mode is oblique and subsonic for non-reacting flow at $M_1 = 5$.

At $T_{ad} = 4$, even though the relative Mach number becomes subsonic with increasing obliquity, the center mode is stabilized by the heat release and only the outer modes are amplified. The latter are inflectional modes whose maximum amplification rates decrease as the obliquity increases. Therefore, heat release makes the dominant mode two-dimensional even in the high Mach number regime. The three-dimensional modes which dominate in the non-reacting case are stable.

Sandham and Reynolds [5] normalized the growth rates by incompressible growth rates at the same velocity and temperature ratios to isolate the effect of compressibility:

$$R(M) = \frac{-\alpha_{i\max}\left(M_c, \bar{u}_2, \bar{T}_2\right)}{-\alpha_{i\max}\left(0, \bar{u}_2, \bar{T}_2\right)} \qquad (13)$$

where M_c is the convective Mach number defined by Papamoschou and Roshko [9]. The results validate use of the isentropic convective Mach number as a compressibility parameter. We plot the normalized two- and three-dimensional growth rates from the current study versus the isentropic convective Mach number, M_c , in Fig. 6 along with experimental data [9-12]. It shows that the present results agree well with the experimental trend that growth rates decrease with increasing convective Mach number, but the large spread in the experimental data prevents a definitive comparison. As expected, the three-dimensional data show better agreement with the experimental data at high Mach numbers than two-dimensional ones. Note that three-dimensional growth rates of instabilities decrease as the Mach number increases, but the experimental data by Papamoschou and Roshko [9] approach an asymptotic value.

4. Conclusions

The three-dimensional instability was studied for the compressible reacting mixing layer. For non-reacting supersonic flows at $M_c > 0.6$, the most unstable modes are oblique center modes that are subsonic relative to both free streams. For reacting flows with $T_{ad} > 3$, the most unstable modes are two-dimensional outer modes even at high Mach numbers. The three-dimensional growth rates normalized by the corresponding incompressible growth rates show good agreement with experimental data rather than the two-dimensional ones.

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